

by linearly extrapolating the stress relation for the two preceding zones:

$$\sigma_{r_i}^I = 2\sigma_{r_{i-1}} - \sigma_{r_{i-2}} \dots \quad (17)$$

$\sigma_{t_i}^I$  and  $\sigma_{a_i}^I$  being derived from two equations of the same type.

Calculation of the second approximation ( $\sigma_{r_i}^{II}, \sigma_{t_i}^{II}, \sigma_{a_i}^{II}$ ) is carried out according to Fig. 9, which gives the layout of a complete iteration. The steps in this iteration are as follows:

(1) The second approximation of  $\sigma_{r_i}$  is derived from equation (12).

(2) The value  $\epsilon_{t_i}$ , imposed for the strain  $\epsilon_i$  by the condition of compatibility, is derived from equation (13); the value  $\epsilon_{a_i}$ , imposed for the strain  $\epsilon_{a_i}$ , is given by the stress field at the outer radius and does not depend on the zone considered.

(3) The strains  $\epsilon_{t_w}$  and  $\epsilon_{a_w}$ , corresponding to the stresses  $\sigma_{r_i}^{II}, \sigma_{t_i}^I$ , and  $\sigma_{a_i}^I$ , are derived; in most cases these strains are not identical with the strains  $\epsilon_{t_i}$  and  $\epsilon_{a_i}$ .

(4) The strains  $\epsilon_{t_a}$  and  $\epsilon_{a_a}$ , corresponding to the stresses  $\sigma_{r_i}^{II}, \sigma_{t_i}^I + \Delta\sigma$ , and  $\sigma_{a_i}^I$  (where  $\Delta\sigma$  is an arbitrary, though relatively small, increment) are derived.

(5) The strains  $\epsilon_{t_b}$ , and  $\epsilon_{a_b}$ , corresponding to the stresses  $\sigma_{r_i}^{II}, \sigma_{t_i}^I$ , and  $\sigma_{a_i}^I + \Delta\sigma$ , are derived.

(6) The quantities  $\alpha$  and  $\beta$ , defined by the following equations, are derived:

$$\alpha = \frac{(\epsilon_{t_w} - \epsilon_{t_i})(\epsilon_{a_b} - \epsilon_{a_w}) - (\epsilon_{a_w} - \epsilon_{a_i})(\epsilon_{t_b} - \epsilon_{t_w})}{(\epsilon_{t_a} - \epsilon_{t_w})(\epsilon_{a_b} - \epsilon_{a_w}) - (\epsilon_{a_a} - \epsilon_{a_w})(\epsilon_{t_b} - \epsilon_{t_w})} \quad (18)$$

$$\beta = \frac{(\epsilon_{a_w} - \epsilon_{a_i})(\epsilon_{t_a} - \epsilon_{t_w}) - (\epsilon_{t_w} - \epsilon_{t_i})(\epsilon_{a_a} - \epsilon_{a_w})}{(\epsilon_{t_a} - \epsilon_{t_w})(\epsilon_{a_b} - \epsilon_{a_w}) - (\epsilon_{a_a} - \epsilon_{a_w})(\epsilon_{t_b} - \epsilon_{t_w})} \quad (19)$$

(7) The second approximation of  $\sigma_{t_i}$  and  $\sigma_{a_i}$  is given by

$$\sigma_{t_i}^{II} = \sigma_{t_i}^I + \alpha \Delta\sigma; \quad \sigma_{a_i}^{II} = \sigma_{a_i}^I + \beta \Delta\sigma \quad (20)$$

(8) The second approximation as a whole ( $\sigma_{r_i}^{II}, \sigma_{t_i}^{II}, \sigma_{a_i}^{II}$ ) is then compared with the first; if they differ, a further iteration is carried out starting from the second approximation, and so on,

*Time required for the calculations*

The calculation method set out above has been programmed for an IBM 1130 computer. The following example will give an indication of the time required: approximately 6 min were taken to carry out a complete calculation in which the cylinder wall was divided into 200 zones and the three stresses  $\sigma_{r_i}, \sigma_{t_i}$ , and  $\sigma_{a_i}$ , were printed for 20 points distributed over the thickness of the wall.

**APPENDIX 2**

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